North Curry C of E Primary School

Calculations Policy

Signed by:

_____ Headteacher Date: _____

_____ Chair of governors Date: _____

Review date: September 2023

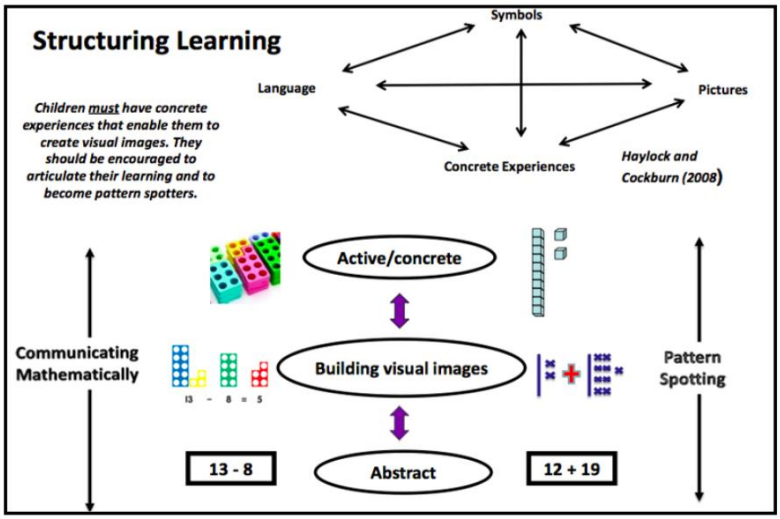
This policy has been designed to teach children to develop conceptual understanding through the progression of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Using the concrete-pictorial-abstract approach:
Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation The enactive stage - a pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

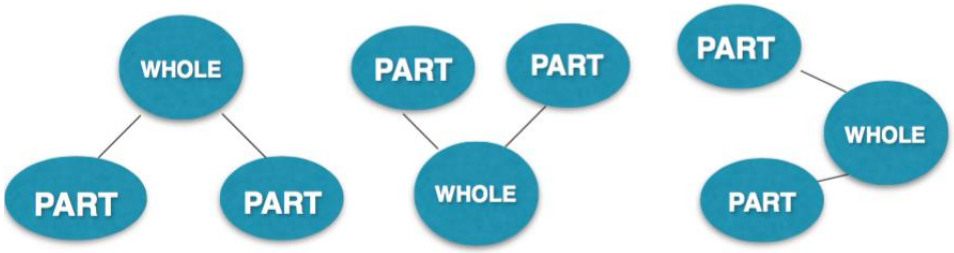
Pictorial representation The iconic stage - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 .



Part/Whole Model – Key Structures

Addition and Subtraction are connected. Add parts together to equal the whole, whole subtract part to name the missing part.



Guidance

This is document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified.

Early Years and KS1 (This includes Nursery)

Developing Number Sense

Vocabulary

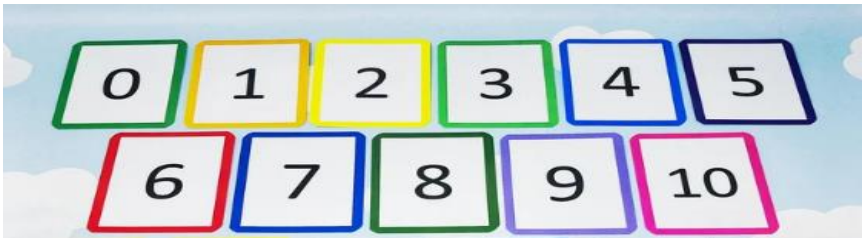
Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? Subitise

Counting

The one to one principle: This involves children assigning one number name to each object that is being counted.

Concrete:	Pictorial:	Abstract:
 Children are able to count a range of objects by touching each one. They may use counting wands to help.	 Children start to count pictures of objects.	 Children start to record the number of items they have counted/ start writing numbers below each item.

The stable-order principle: Children understand when counting, the numbers have to be said in a certain order.



The Cardinal principle: Children understand that the number name assigned to the final object in the group is the total numbers of objects in that group.

Concrete:	Pictorial:	Abstract:
Children use a range of structured and unstructured apparatus, plus natural 	Children recognize different number values that are represented in pictorial forms. 	Children are asked a range of questions that allow them to show an application understanding related to cardinality. E.g. Can you find me a collection of…(objects) to represent four? 

<u>The abstraction principle</u>: This is the understanding that anything can be counted, not just things that we can touch. For example claps, jumps, beats of music.

<u>The order-irrelevance principle:</u> This is the understanding that things can be counted in any order but will still give us the same total.

<u>Concrete:</u>	**<u>Pictorial:</u>**	**<u>Abstract:</u>**
Children to be given objects they can count putting them in different orders and where the teacher will put them in different arrangements.	Children given pictures of objects that they can count in any order. They may want to mark off with a pen to show they have counted them.	Children to draw their own pictures and show how they many count them in different ways.

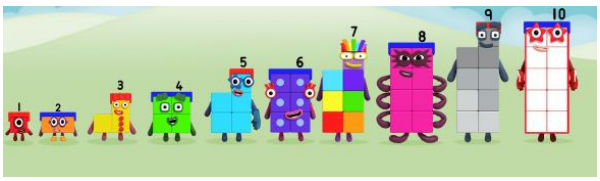
		

Subitising:

Concrete:	Pictorial:	Abstract:
Children replicate a range of physical representations, which they can verbally interpret without a need to count objects.  	Children use picture prompts to practice their recognition of number representation.  	Children can use different ways of showing various representations of numbers to 5. 

Odd and even:

Children understand the concept of odd and even. Representations of numicon and cubes is good for seeing the even and uneven top to encourage discussions and sorting of numbers.

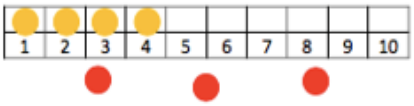
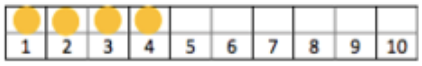


Concept of zero:

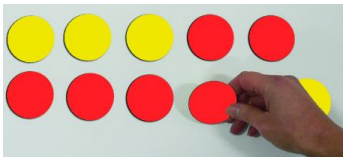
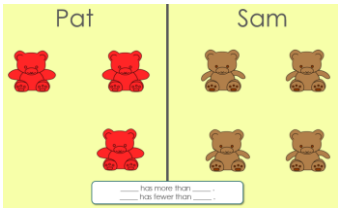
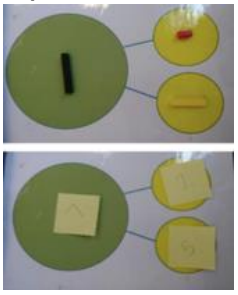
Concrete:	Pictorial:	Abstract:
Looking at items on a plate noticing how many are on a plate and talking about where there is nothing, this is the number 0.  	Children use pictorial representations to see that you can have an amount that's called 'zero'. Pupils are required to count the number of apples on a tree, and circle the trees which have no apples.	Children will be encouraged to represent written number sentences by using equipment to support where there is nothing to add to something. Eg 5=5+0.

Counting on:

Concrete:	Pictorial:	Abstract:
Children use physical objects to learn the skill. They count from the larger value and count on the other value, they may use their hands to do this or count the objects they have.  They can also use number track to rehearse the process of counting on. 	Children use a die to generate numbers and count on from pictorial representations. They may also use their skills of subitising and then count on from there**.** 	Children apply their understanding of this skill by playing games such as snakes and ladders. 

Equality:

Concrete:	Pictorial:	Abstract:
Children use physical equipment when learning about equality. They use the language of: the same as, more, fewer as well as equal to. 	Children can use pictorial representations to show equality and are encouraged to verbalise their reasoning. 	Children use the cherry model to record either written numerals or pictorial representations that highlight the concept of 'the same as…' 

Conservation of number:

Being able to explore and have the understanding that when objects are moved from place to place there is still the same total. This can be explored by moving dolls between different rooms, having speckled frogs jumping from the log into the water.

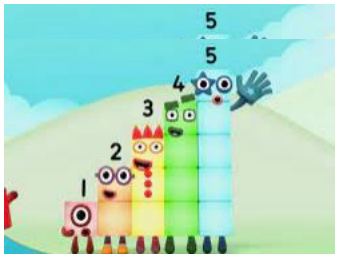
Addition

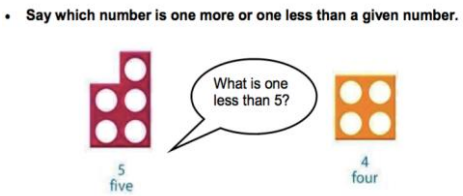
Vocabulary

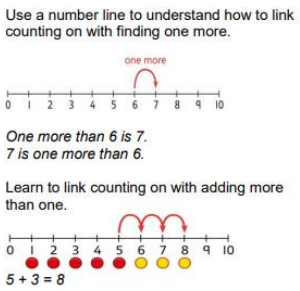
Part, whole, add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on.

One more and one less:

Using the staircase pattern supports children with seeing and understanding 1 more and 1 less.

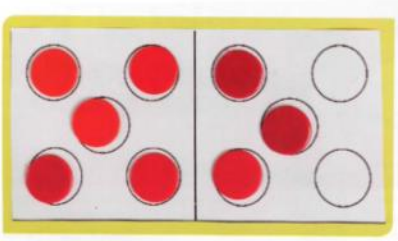






Knowing and exploring number facts:

Children to have a good understanding of the number facts up to 10. Children to use fingers, 5 and 10 frames as well as Hungarian 10 frames to support in this.

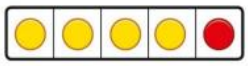


Break apart a group and put back together to find and form number bonds.

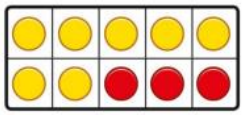
3 + 4 = 7

6 = 2 + 4

Use five and ten frames to represent key number bonds.



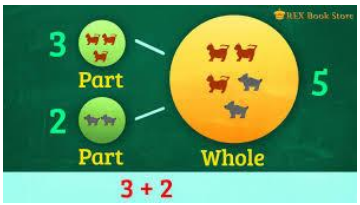
5 = 4 + 1



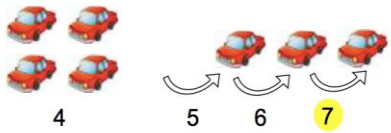
10 = 7 + 3

Explore part /whole relationship:

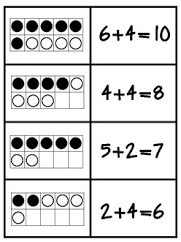
This may happen through stories and songs such as 5 little specked frogs- looking at the parts and the whole.

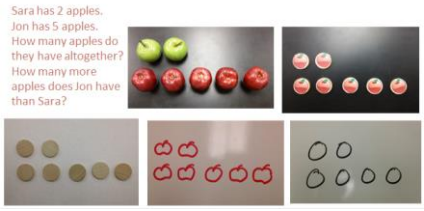


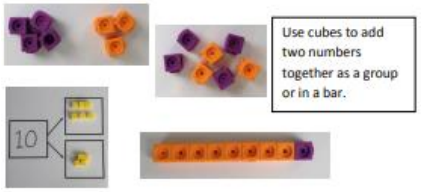
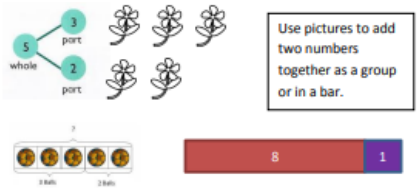
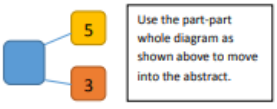
- **Use objects to add two single-digit numbers by counting on to find the answer.**



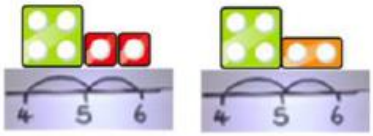
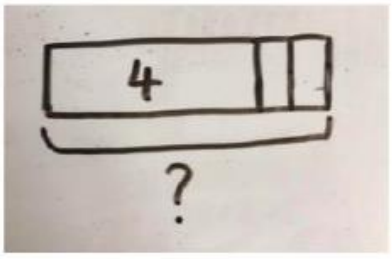
4 + 3 = 7



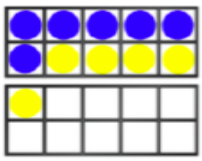


Concrete:	Pictorial:	Abstract:
Combining two parts to make a whole. 		 **4 + 3 = 7** **10= 6 + 4**

Counting on

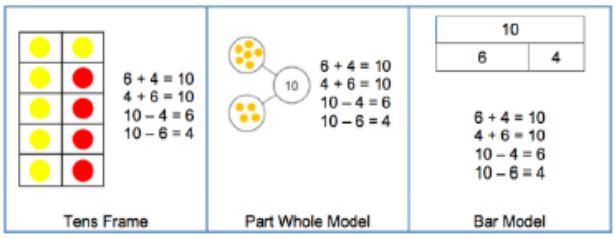
Concrete:	Pictorial:	Abstract:
 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Counting on using a number line using cubes or numicon to support.  	Children to represent their counting on using a bar. 	Children to use number lines and start to draw their own to find the sum of two numbers. 

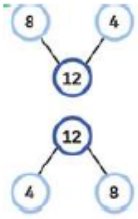
Regrouping to make 10

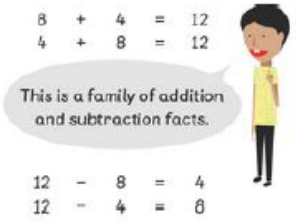
Concrete:	Pictorial:	Abstract:
 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	Children draw counters on 10 frames to support their thinking. 	Children to develop an understanding of equality. **6+□ = 11** **6+5= 5+ □** **6+5= □ +4**

Learning number bonds to 20 and demonstrate the related facts:

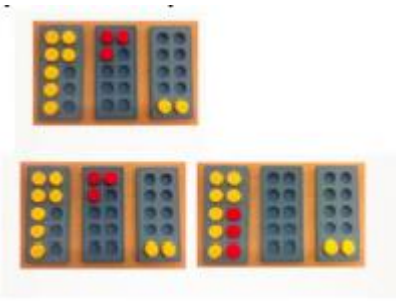
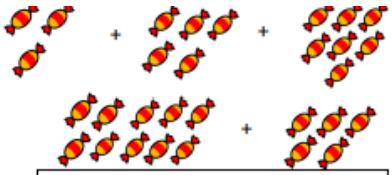
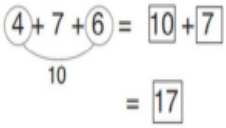
Teach addition and subtraction alongside each other as pupils need to see the relationship between the facts.





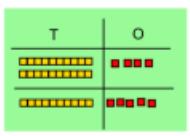
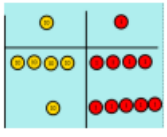
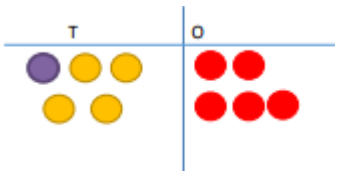
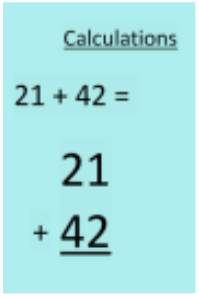


Adding three single digits together:

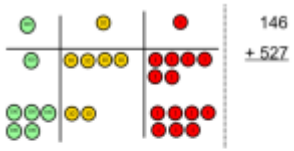
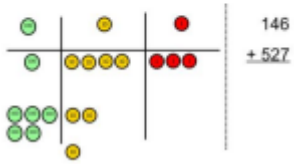
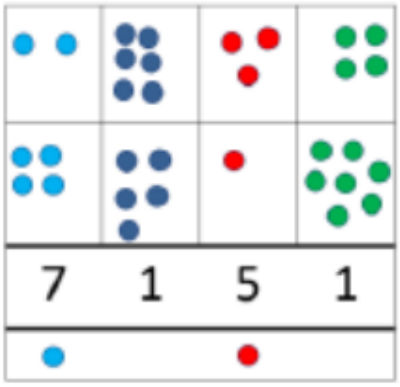
Concrete:	Pictorial:	Abstract:
4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.   **7+3+2= leads to 10+2=**	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

Using the column to add:

The column method without re-grouping.

Concrete:	Pictorial:	Abstract:
Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	 Calculations 21 + 42 = 21 + 42

Using the column and regrouping:

Concrete:	Pictorial:	Abstract:
Make both numbers on a place value grid.   Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw their own version or learn to read the pictorial versions of the column.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 11 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

Subtraction

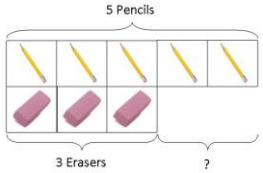
Vocabulary

Part, whole, equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

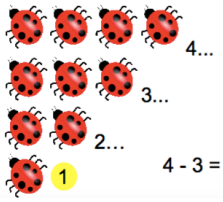
- **Use objects to subtract two single-digit numbers by counting back to find the answer.**

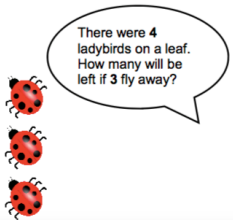
The first step into **subtraction** is to learn how to count **backwards**.

8 − 4 = __



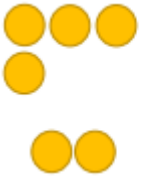
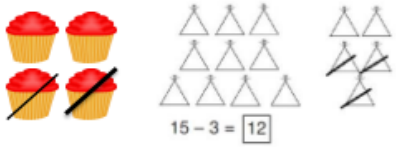
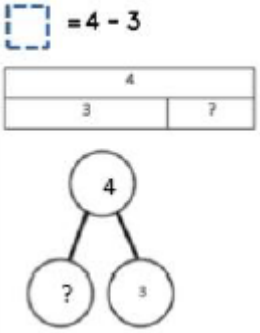
Children will then utilise this strategy to solve simple subtractions:



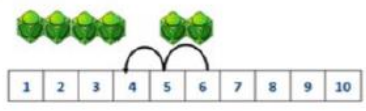
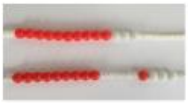
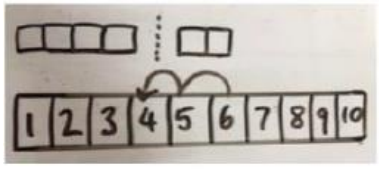
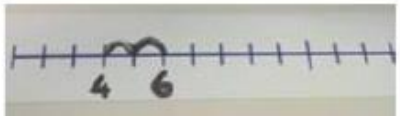


Solving problems using concrete and pictorial images.

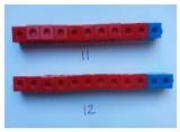
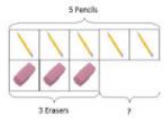
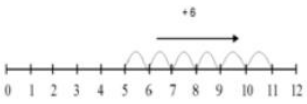
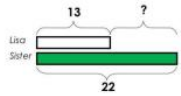
Physically taking away and removing objects from a whole:

Concrete:	Pictorial:	Abstract:
Five and ten frames, numicon, cubes and other objects can be used.  6 – 2 = 4	Children to use pictures and cross them out if they are taking them away and for children to progress 	4- 3 = 

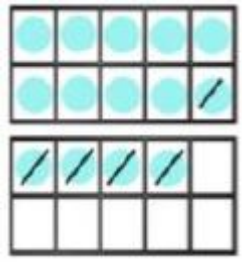
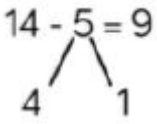
Subtraction by counting back:

Concrete:	Pictorial:	Abstract:
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4  13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

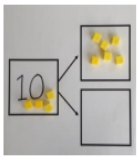
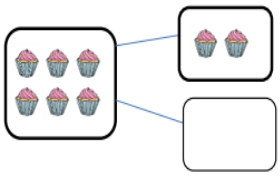
Finding the difference:

Concrete	Pictorial:	Abstract:
Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference.	 Count on to find the difference. **Comparison Bar Models** Draw bars to find the difference between 2 numbers. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Find the difference between 8 and 5. 8-5, the difference is Children to explore why 9-6=8-5=7-4 have the same difference.

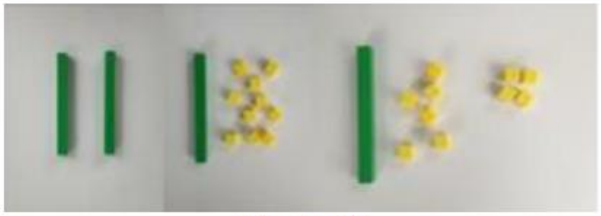
Making 10:

Concrete:	Pictorial:	Abstract:
14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	Children to present the ten frame pictorally and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 - 4 = 10 10 - 1 = 9

Using the whole part model:

Concrete:	Pictorial:	Abstract:
 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? **10 - 6 =**	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

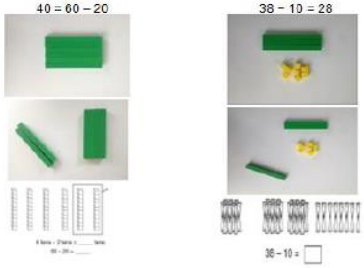
When subtracting using Dienes children should be taught to regroup a ten rod for 10 ones and then subtract from those ones



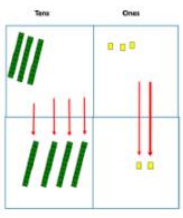
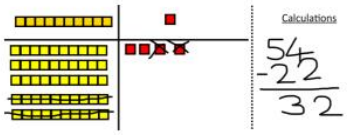
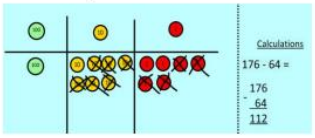
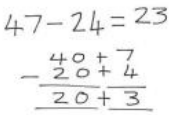
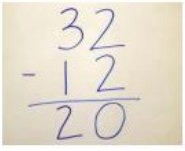
20 – 4 = 16

Subtracting multiples of 10

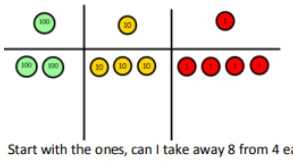
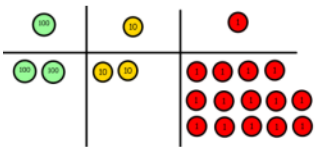
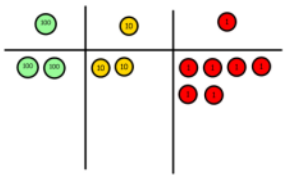
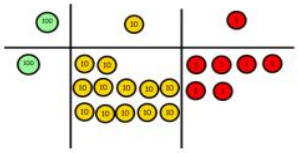
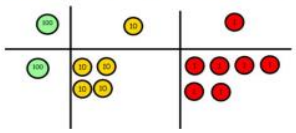
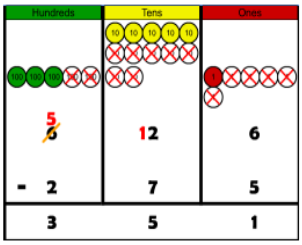
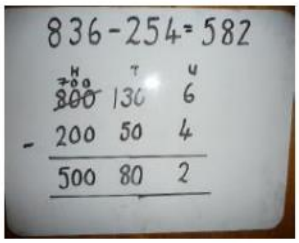
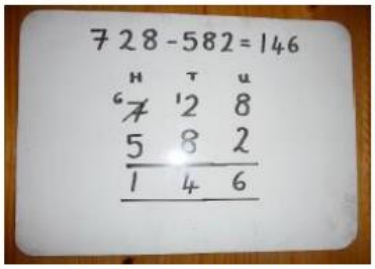
Using the vocabulary of 1 ten, 2 tens etc alongside 10, 20, 30 Is very important here as pupils need to understand that it is a 10 not a 1 that is being taken away



Column method without re-grouping:

Concrete:	Pictorial:	Abstract:
 Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first.	Draw the Base 10 or place value counters alongside the written calculation to help to show working.  	 This will lead to a clear written column subtraction. 

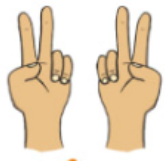
Column method with re-grouping:

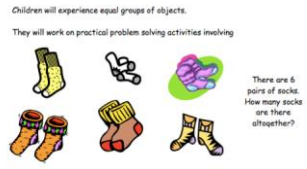
Concrete:	Pictorial:	Abstract:
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88 Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations 224 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5

Multiplication

Vocabulary

Part, whole, groups of, lots of, doubles



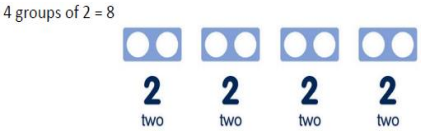


2 and 2 is 4. Double 2 is 4.

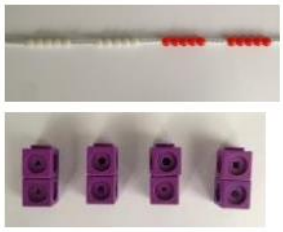
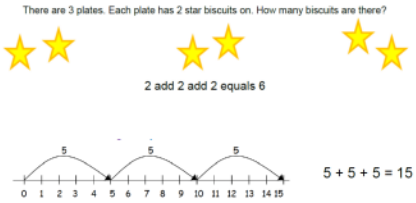
Children will use a range of objects in the tens frames, their fingers and then progress to drawing their own doubles.

Counting in multiples/ skip counting:

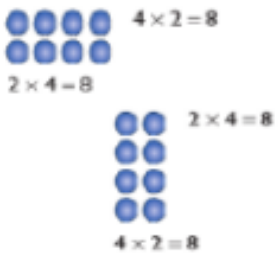
Concrete:	Pictorial:	Abstract:

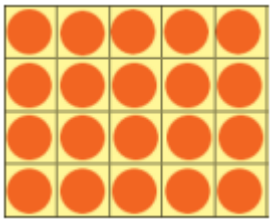
  Lots of counting in multiples. Children should count the number of groups on their fingers as they are skip counting.	When moving to pictorial/written calculations the vocabulary is important.   This image represents two groups of 4 or 4 twice	Children to record their counting **2, 4, 6, 8…** **5, 10, 15, 20…**

Repeated addition:

Concrete:	Pictorial:	Abstract:
 Count in multiples supported by concrete objects in equal groups.	 	Write addition number sentences to match the pictures. 

Arrays- showing commutative multiplication:

Concrete:	Pictorial:	Abstract:
Create arrays using counters/ cubes to show multiplication sentences.	Draw arrays in different rotations to find commutative multiplication sentences.  	Use an array to write multiplication sentences and reinforce repeated addition.

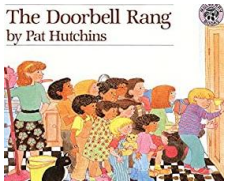
 		 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

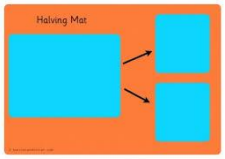
Division

Vocabulary

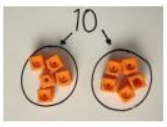
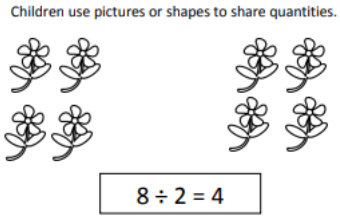
Part, whole, share, share equally, one each, two each…, group, groups of, lots of.

Children will learn about division through stories and practical activities. They will be sharing and grouping items to help them. They will be encouraged to make connections with t
heir doubles work.



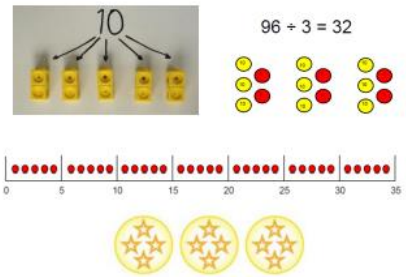
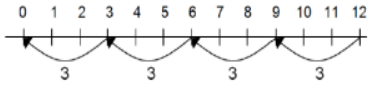
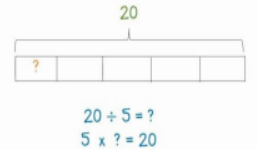


Sharing objects into groups:

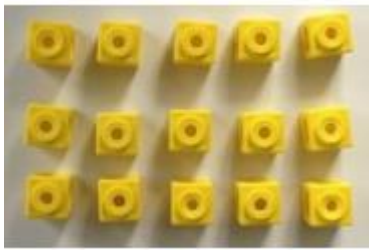
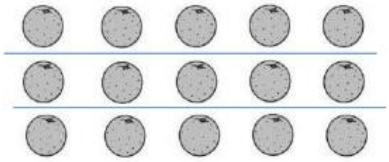
Concrete:	Pictorial:	Abstract:
   I have 10 cubes, can you share them equally in 2 groups?	 Children use pictures or shapes to share quantities. 8 ÷ 2 = 4	Children start to solve problems using their number fact knowledge and the resources they feel confident with. **Share 9 buns between three people. 9 ÷ 3 = 3**

Division as grouping:

Concrete:	Pictorial:	Abstract:
Divide quantities into equal groups. Use cubes, counters, objects or place value		Divide 28 into 7 groups. How many are in each group?

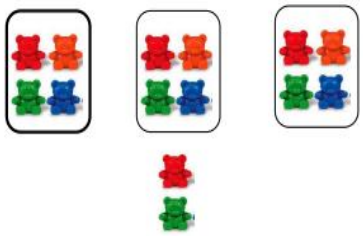
counters to aid understanding. 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4

Division with arrays:

Concrete:	Pictorial:	Abstract:
Link division to multiplication by creating an array and thinking about the number sentences that can be created.  E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. **7 x 4 = 28** **4 x 7 = 28** **28 ÷ 7 = 4** **28 ÷ 4 = 7**

Division with a remainders:

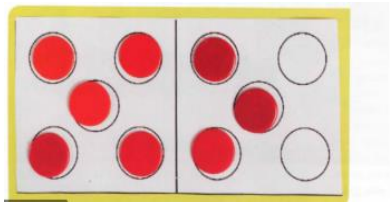
Concrete:	Pictorial:	Abstract:
	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.	Complete written divisions and show the remainder using r.

14 ÷ 3 = Divide objects between groups and see how much is left over 	 Draw dots and group them to divide an amount and clearly show a remainder.  	29 ÷ 8 = 3 REMAINDER 5 dividend divisor quotient remainder

Resource list:

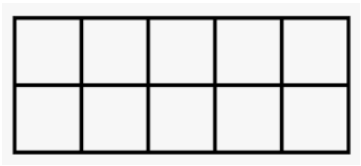
This is a list of equipment that you may want to use within your teaching to best support the children.

- Numicon and numicon boards
- Feely bags
- Number tracks or number lines
- Counters and double sided counters
- Standard dice
- Number balance
- Paper plates
- Counting objects



- Hungarian ten frames

Interlocking cubes



Tens frames

- Rekenrecks

Cuisenaire rods

Pattern blocks